DAFNY

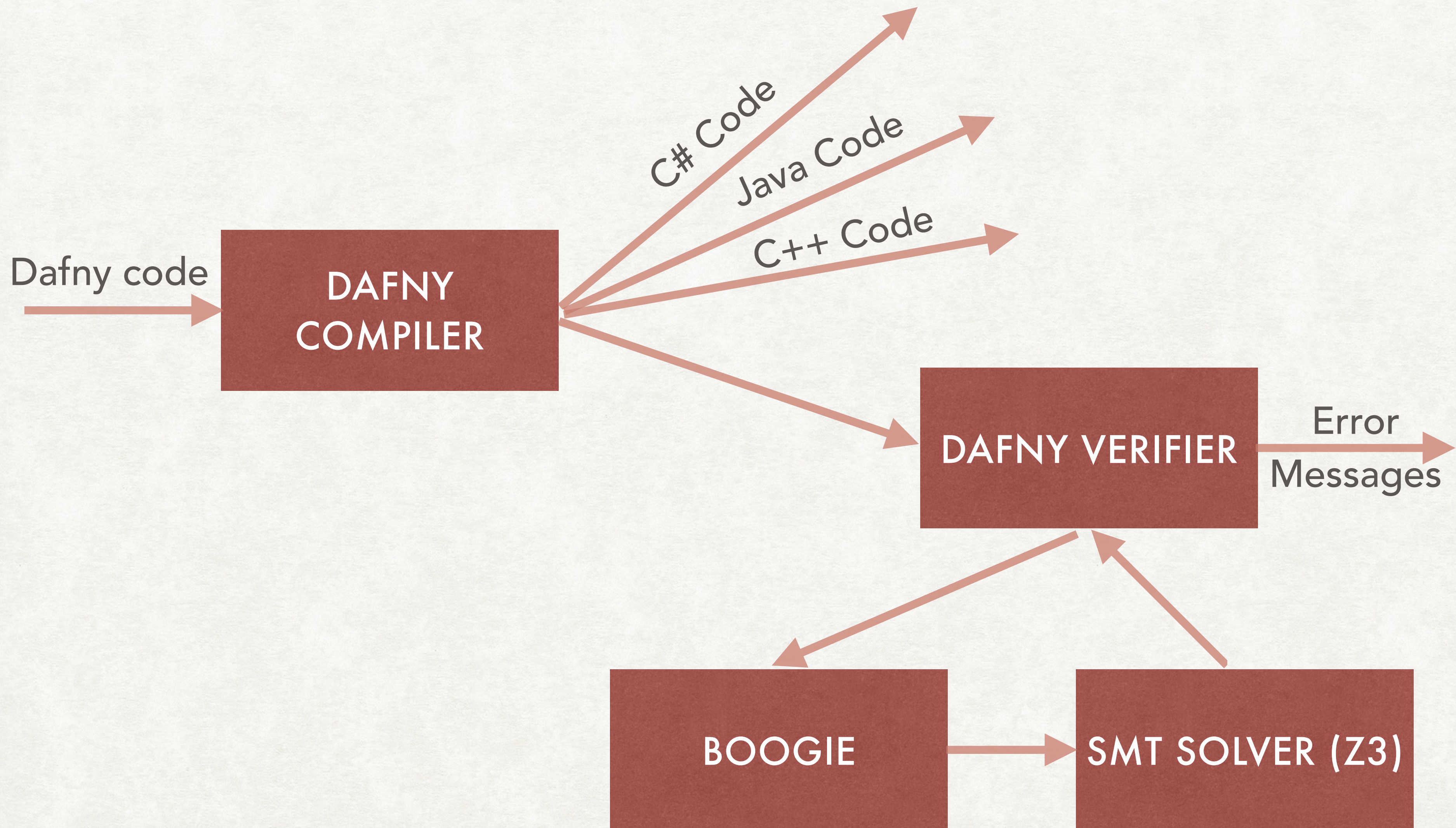
INTRODUCTION

- Dafny is a programming language, verifier and compiler, all rolled together into one.
- Dafny uses SMT solver (Z3) to automatically prove correctness.
- Highly sophisticated specification language
    - Pre-conditions and Post-conditions for methods
    - Loop Invariants
    - Assertions
    - Lemmas and Theorems

DAFNY - PROGRAMMING LANGUAGE

- Dafny hosts two separate sub-languages
- Language for the executable code
  - Imperative, Object-oriented
  - Methods, loops, if-then-else, arrays, classes, inductive data-types,…
- Specification Language
  - Functional
  - Pure functions, predicates, algebraic datatypes, sets, sequences, lemmas…

DAFNY TOOL



DAFNY BASICS: METHODS

- Parameters and return values must be typed.
- Return values are named.
- Multiple return values are also allowed.

```
method Abs(x: int) returns (y: int)
{
    ...
}
```

```
method MultipleReturns(x: int, y: int)
returns (more: int, less: int)
{
    ...
}
```

DAFNY BASICS: METHODS

```
method MultipleReturns(x: int, y: int)
returns (more: int, less: int)
{
   more := x + y;
   less := x - y;
}
```

- To return a value, assign it to the named return variable.
  - In case of a single return value, the ‘return’ statement can also be used.
- Assignments use ‘:=’.
- Compound statements such as if..then..else, while loops are available.

DAFNY BASICS: PRE AND POST-CONDITIONS

```
method MultipleReturns(x: int, y: int)
returns (more: int, less: int)
   ensures less < x
   ensures x < more
{
   more := x + y;
   less := x - y;
}
```

- Post-conditions specified using ‘ensures’.
  - Are the post-conditions satisfied in the above example?

DAFNY BASICS: PRE AND POST-CONDITIONS

```
method MultipleReturns(x: int, y: int)
returns (more: int, less: int)
   requires y > 0
   ensures less < x
   ensures x < more
{
   more := x + y;
   less := x - y;
}
```

- Post-conditions specified using ‘ensures’.
  - Are the post-conditions satisfied in the above example?
- Pre-conditions specified using ‘requires’
  - Dafny will output ‘Verified’

DAFNY BASICS: ASSERTIONS

- Local variables declared using ‘var’.
- Methods calls are not allowed inside assert conditions.
  - Will the assert in ‘Testing’ method succeed?

```
method Abs(x: int) returns (y: int)
   ensures y >= 0
{
   if x < 0
     { return -x; }
   else
     { return x; }
}
method Testing()
{
   var v := Abs(3);
   assert v == 3;
}
```

DAFNY BASICS: ASSERTIONS

- Local variables declared using 'var'.
- Methods calls are not allowed inside assert conditions.
  - Will the assert in 'Testing' method succeed?

```
method Abs(x: int) returns (y: int)
   ensures y >= 0
   ensures x >= 0 ==> y == x
   ensures x < 0 ==> y == -x
{
   if x < 0
     { return -x; }
   else
     { return x; }
}
method Testing()
{
   var v := Abs(3);
   assert v == 3;
}
```

DAFNY BASICS: FUNCTIONS

```
function abs(x: int): int
{
   if x < 0 then -x else x
}
```

- Pure mathematical functions
  - Cannot write to memory
  - Body is a single expression
  - Single return value
  - Not compiled and executed
- Can only be used inside pre-conditions, post-conditions, assertions and loop invariants

DAFNY BASICS: FUNCTIONS

```
function abs(x: int): int
{
   if x < 0 then -x else x
}
method Abs(x: int) returns (y: int)
   ensures y == abs(x)
{
   if x < 0
     { return -x; }
   else
     { return x; }
}
method Testing()
{
   var v := Abs(3);
   assert v == 3;
}
```

DAFNY BASICS: LOOP INVARIANTS

```
var i := 0;
   while i < n
      invariant 0 <= i
   {
      i := i + 1;
   }
```

- Specified using the keyword ‘invariant’
  - Must hold upon entering the loop
  - After every iteration of the loop body
  - Dafny will automatically check the conditions

DAFNY BASICS: LOOP INVARIANTS

```
method ComputeFib(n: nat) returns (b: nat)
   ensures b == fib(n)
{
   if n == 0 { return 0; }
   var i: int := 1;
   var a := 0;
       b := 1;
   while i < n
      invariant 0 < i <= n
      invariant a == fib(i - 1)
      invariant b == fib(i)
   {
      a, b := b, a + b;
      i := i + 1;
   }
}
```

```
function fib(n: nat): nat
{
   if n == 0 then 0 else
   if n == 1 then 1 else
      fib(n - 1) + fib(n - 2)
}
```

DAFNY BASICS: TERMINATION

- Dafny will try to prove that every loop terminates
  - Reports an error if it can't prove termination.
- Dafny will guess an expression which decreases with every iteration and has a lower bound.
  - This can be explicitly specified using the 'decreases' annotation.
  - Use the annotation 'decreases *' if we don't want Dafny to prove termination.

```
while 0 < i
     invariant 0 <= i
     decreases i
  {
     i := i - 1;
  }
```

DAFNY BASICS: TERMINATION

- Dafny will try to prove that every loop terminates
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- Dafny will guess an expression which decreases with every iteration and has a lower bound.
  - This can be explicitly specified using the 'decreases' annotation.
  - Use the annotation 'decreases *' if we don't want Dafny to prove termination.

```
while 0 < i
      invariant 0 <= i
      decreases i
   {
      i := i - 1;
   }
```

```
while i < n
      invariant 0 <= i <= n
      decreases n - i
   {
     i := i + 1;
   }
```

DAFNY BASICS: ARRAYS

- Built in data type ‘array<T>’
    - Also built in field ‘Length’.
- Dafny will automatically check that arrays are accessed within the bounds.

```
method M(x: int)
{
   var a := new int[10];
   var b := a[x];  //Error
   if 0 <= x < 10
     { b := a[x];  //OK }
}
method M(x: int, c: array<int>)
 requires 0 <= x < 10
{
   var a := new int[10];
   var b := a[x];  //OK
   If 0 <= x < 10
     { b := c[x];  //Error }
}
```

DAFNY BASICS: QUANTIFIERS

```
method Find(a: array<int>, key: int) returns (index: int)
   ensures 0 <= index ==> index < a.Length && a[index] == key
   ensures index < 0 ==> forall k :: 0 <= k < a.Length ==> a[k] != key
{
   index := 0;
   while index < a.Length
      invariant 0 <= index <= a.Length
      invariant forall k :: 0 <= k < index ==> a[k] != key
   {
      if a[index] == key { return; }
      index := index + 1;
   }
   index := -1;
}
```

- Quantifiers can be used in pre/post-conditions, assertions, invariants using the ‘forall’ keyword.

DAFNY BASICS: LEMMAS

```
method ComputePow2(n: nat) returns (p:nat)
 ensures p = pow2(n)
{
  if n = 0
    { p := 1; }
  else if n % 2 = 0
    { p := ComputePow2(n / 2);
      p := p * p; }
  else
    { p := ComputePow2(n-1);
      p := 2 * p;
    }
}
```

```
function pow2(n: int): int
 requires 0 <= n
{
    if n == 0 then 1
  else
    2*pow2(n-1)
}
```

DAFNY BASICS: LEMMAS

```
method ComputePow2(n: nat) returns (p:nat)
 ensures p = pow2(n)
{
  if n = 0
    { p := 1; }
  else if n % 2 = 0
    { p := ComputePow2(n / 2);
      p := p * p;
      Lemma(n); }
  else
    { p := ComputePow2(n-1);
      p := 2 * p;
    }
}
```

```
function pow2(n: int): int
 requires 0 <= n
{
    if n == 0 then 1
  else
    2*pow2(n-1)
}
```

```
lemma Lemma(n : nat)
   requires n % 2 == 0
   ensures pow2(n) == pow2(n/2)
* pow2(n/2)
{
   if n != 0 then Lemma(n-2);
}
```